13 October 2021

Problem Solving Club

Using Algebra

Using Algebra

Algebra is the manipulation of variables.

Say there were 120 tickets sold for a concert and $2200 of revenue was made. Child tickets cost $10 and adult tickets cost $20. How many of each ticket were sold?

Let c be the number of child tickets.
Let a be the number of adult tickets.

$c + a = 120$

$c = 120 - a$

$10c + 20a = 1530$

$10(120 - a) + 20a = 1530$

$a = 33$ $c = 87$

Using Algebra

Some problems have explicit algebraic parts
Other problems are well-suited to
introducing algebra.

ex. Pumpkin Problem
School Trip Problem

How do we know when to introduce algebra?

- Relationship between quantities are given
- The quantities asked for don't seem directly related to the information

Using algebra is a good technique if you are someone who defaults to using trial and error

Using Algebra

THE DANGERS OF ALGEBRA

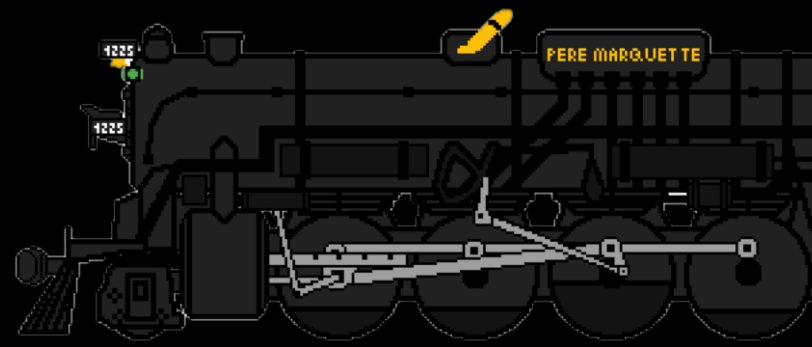
Introducing too many variables and over complicating the problem

Missing a simpler non-algebraic solution

Overcomplicated Train Problem

v km/h

40 km/h



b

3/8

d

Overcomplicated Train Problem

(1) $(3/8)b/v = d/40$

(2) $(5/8)b/v = (d+b)/40$

(1) $(3/8)40/v = d/b$

$15/v = d/b$

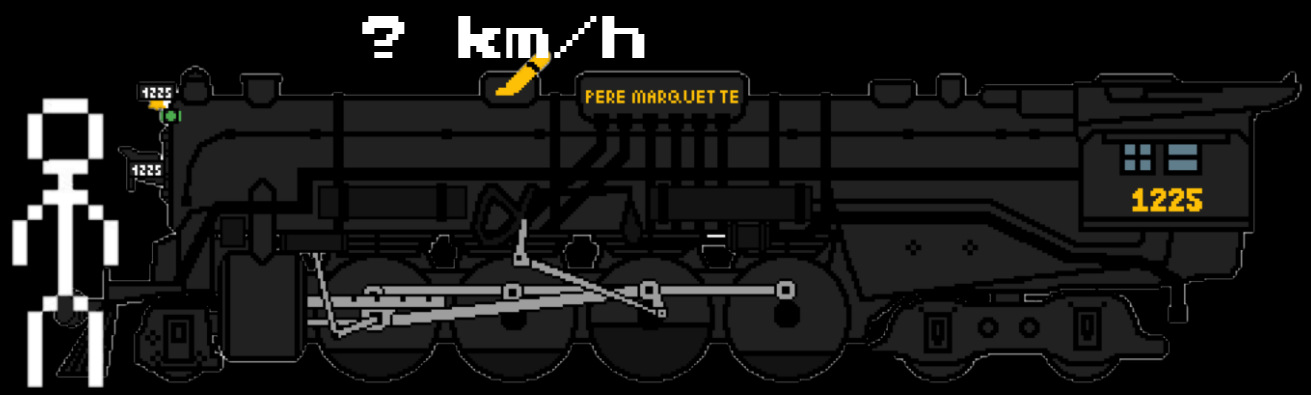
(2) $(5/8)40/v = (d+b)/b$

$25/v = d/b + 1$

$25/v - 1 = d/b$

$15/v = 25/v - 1 \quad \rightarrow \quad v = 10$.

Simple Train Problem



40 km/h

∴ We must travel 4 times as slow as the train.

Our speed = 40/4 = 10 km/h

Arithmetic Sequences

An arithmetic sequence is a sequence with a constant difference.

2 5 8 11 14 17 20 23...

An arithmetic sequence can be defined by an initial value a and a difference d.

a a+d a+2d a+3d a+4d a+5d...

2 2+3 2+2(3) 2+3(3) 2+4(3)...

14 − 11 = 11 − 8 = 3

Geometric Sequences

A geometric sequence is a sequence with a constant ratio between terms.

3 6 12 24 48 96 192...

A geometric sequence can be defined by an initial value a and a ratio r.

$a \quad ar \quad ar^2 \quad ar^3 \quad ar^4 \quad ar^5 \quad ar^6 ...$

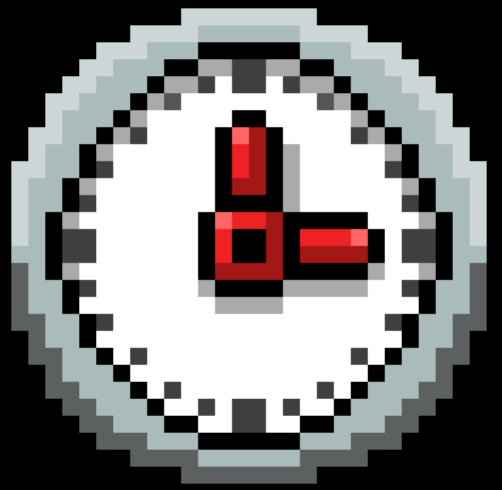
$3 \quad 3(2) \quad 3(2)^2 \quad 3(2)^3 ...$

$48/24 = 24/12 = 2$

Week 3 — Oct 13

1. Alexandra drives on a straight road from Toronto (T) to Mississauga (M) every day. Some of this road is uphill, some is flat, and some is downhill. She can travel uphill at 63 km/h, on flat ground at 77 km/h, and downhill at 99km/h. If it takes her 3h 40min to travel from T to M and 4h 20min to travel from M to T, what is the distance between M and T?

2. Brandon started researching keyboards between 9pm and 10pm. When he finished between 10pm and 11pm on the same night, the hour hand was exactly where the minute hand was when he started, and the minute hand was exactly where the hour hand was when he started. He spent t hours researching. What is t?



3. Challenging Problem!!
a, b, and c are 3 different integers that satisfy the following conditions:
- $abc = 17955$
- a, b, and c form an arithmetic sequence in that order
- $(3a+b)$, $(3b+c)$, and $(3c+a)$ form a geometric sequence in that order.

What is $a+b+c$?