Week 11 - Dec 15

Here are the 3 problems for the Winter Break. Have Fun!

1 - Russian Multiplication

To multiply two numbers together, Russians start by writing the two numbers next to each other. They repeatedly divide the left number by 2 until they reach 1, writing down the numbers they get as they go. When they encounter a .5 at the end of a number, they ignore it because Russians hate fractions. Next, they double the number in the right column until the two columns are the same length.

The only thing Russians hate more than fractions is even numbers in the left column, so they cross out the entire row when they see one. Finally, they add the remaining numbers in the right column together, and that is the product of the two numbers.

1 – Russian Multiplication

Example: Let's do 35 × 26.

35	26

1 — Russian Multiplication

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35/2 is 17.5, but we don't write the .5 because Russians hate fractions.

35	26
17	

1 – Russian Multiplication

Example: Let's do 35 × 26.

Keep dividing by 2 until we hit 1.	35	26
	17	
	8	
	4	
	2	
	1	

1 – Russian Multiplication

Example: Let's do 35 × 26.

Now keep doubling the number in the right column.

35	26
17	52
8	104
4	208
2	416
1	832

1 — Russian Multiplication

Example: Let's do 35 × 26.

Russians hate even numbers in the left column, so cross all those out.

35	26
17	52
8	104
4	208
2	416
1	832

1 — Russian Multiplication

Example: Let's do 35 × 26.

Now add the remaining numbers in the right column to get our final answer.

35	26
17	52
8	104
4	208
2	416
1	832

26 + 52 + 832
= 910

1 - Russian Multiplication

The problem is: Can you explain why Russian Multiplication works? If you can, try writing a proof of it.

2 — Sum of Subsets

Let's make a set called A.

$A = \{1, 2\}$

A subset of a set is a set where all elements of the subset are also elements of the original set. We can write the set of all the subsets of A, Here they are:

$subsets(A) = \{\{1\}, \{2\}, \{1, 2\}, \{\}\}$

Note that A and the set containing nothing are both subsets of A. We now define the sum of subsets of a set as the sum of all elements of all the subsets of the set. The sum of subsets of A is:

$sumOfSubsets(A) = 1 + 2 + 1 + 2 = 6$

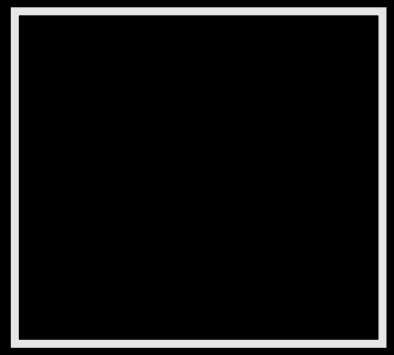
2 - Sum of Subsets

The problem is: What is

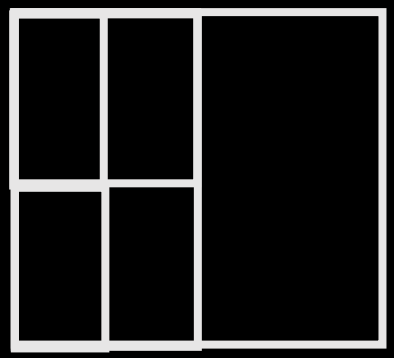
`sumOfSubsets({1, 2, 3, 4, 5, 6, 7, 8, 9, 10})`?

3 — Rectangular Tiling

We want to tile a square with rectangles. That is, we want to completely fill it, so that there are no gaps. The only restriction is that the rectangles' sides have to be in a 2:1 ratio. The problem is: for which numbers of rectangles can we do this?



We clearly can't do it in 1 rectangle.



But we can do it in 5.